The Depth and Breadth of the Step-by-step Innovation Framework*†

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Abstract

In Schumpeterian growth theory, the step-by-step innovation framework has a unique place with its realistic modeling of competition among firms and the effect of firms' strategic behavior on innovation outcomes. As such, this framework provides a fertile ground for examining the basic mechanisms that link competition and innovation and how their interaction shapes firm dynamics and economic growth. In this piece, I summarize two recent papers building on this framework, Akcigit and Ates (2021) and Akcigit et al. (2021), which shed light on the slowing business dynamism in the U.S. economy and the implications of trade and industrial policy in open economies. Doing so, I will highlight key mechanisms (*the depth*) as well as the wide-reaching applications (*the breadth*) of the step-by-step innovation model.

Keywords: Step-by-step innovation, competition business dynamism, international trade.

JEL Classifications: O1, L1, F4, F6

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1 Introduction

Looking back at the past 30 years, one can comfortably conclude that Aghion and Howitt (1992), with the tremendous body of work that followed, was a big bang in the literature of economic growth. The way the theory operationalized the Schumpeterian paradigm and the process of creative destruction did not just reshape how economists think about economic growth but also contributed to the analysis of a vast set of topics, ranging from intellectual property rights policies to international trade to environmental issues. One such fundamental topic is the relationship between competition between firms and economic growth, and the authors contributed to it with another seminal framework that extends the Shumpeterian paradigm in substantial ways: the step-by-step innovation model. In this piece, I will discuss the significance of this—arguably underutilized—framework along with some recent applications.

Notwithstanding its numerous merits, the first-generation model of Schumpeterian creative destruction laid out in Aghion and Howitt (1992) did not truly capture the essence of competition between firms. While innovations are still competitive in that framework—the entrepreneur with an invention replaces the incumbent—it is not the case that firms respond to each other's actions in a forward–looking fashion.[1] Only the outsiders (potential entrants) innovate in equilibrium, whereas incumbents did not, reflecting Arrow's replacement effect. Moreover, the first-generation models predicted only a negative relationship between competition and economic growth predicated on the appropriability effect. Yet subsequent empirical evidence pointed to the other direction, showing that competition can in fact boost economic growth and innovation in various environments.

Faced with these challenges, Philippe Aghion, Peter Howitt, and coauthors strove to build a more flexible framework that can speak to the nature of competition between firms and matches the relevant empirical evidence better. Crucially, they relaxed the assumption of a linear R&D production function and introduced decreasing returns to R&D at the firm level.[2] This small but fundamental change implies that incumbent firms find it optimal to invest in R&D. As introduced and analyzed extensively in a series of seminal papers (Aghion et al., 1997, 2001, 2005), this new feature altered the dynamics of the model greatly while still retaining the core of the Schumpeterian creative destruction.[3]

[1]Still, the concept of competitive innovations and Schumpeterian creative destruction distinguishes these first-generation models from contemporary work on endogenous growth theory. For instance, another milestone in this literature, Romer (1990), conceptualizes the endogenous nature of economic growth through the expansion of varieties in an economy enabled by the creation of non-rivalrous ideas. In this setting, each variety is produced by a monopolist that owns the blueprint for production of that specific variety, and once the monopolist obtains that blueprint, it does not face any competitive pressure from other firms, be it potential entrants with newer blueprints or other incumbents.

[2]The assumption of a linear R&D production function in the earlier vintages of the Schumpeterian creative destruction models is the key reason why incumbent firms refrain from engaging in R&D.

[3]Innovation by incumbent firms is an equilibrium outcome also in another seminal contribution to the literature on firm dynamics, Klette and Kortum (2004), which itself builds on Aghion and Howitt (1992). Yet the nature of incumbent innovation in this setting is horizontal, akin to the expanding–varieties models. Randomly landing on the product of another firm, incumbent innovations are still competitive and generate creative destruction, in a fashion

To begin, the key agents in an industry are two forward-looking incumbent firms that are in a constant competitive race, as opposed to outside entrepreneurs in the benchmark model. As such, the model captures the strategic interaction between a market leader and its competitors in a succinct and explicit way. In this regard, the model has the essence of earlier R&D–race frameworks in the industrial organization literature (Harris and Vickers, 1985, 1987; Grossman and Shapiro, 1987; Budd et al., 1993) and embeds it into a dynamic macro general equilibrium setting. Second, the benefit of innovation is the incremental gain in profits as opposed to the absolute level in the benchmark model, and the size of the increment depends on the relative position of the rivals in the competitive race. Consequently, variables such as markups and market concentration become endogenous outcomes of the model, making the framework very suitable for the analysis of widely-debated economic issues such as market competition and business dynamism. In addition, the model generates a non-linear relationship between competition and economic growth. Again, as in Harris and Vickers, 1987 and Budd et al., 1993, firms' R&D efforts intensify when the technological gap between them decreases in the model—i.e., when competition strengthens—as the incremental gain of innovating, helping firms escape competition, increases. This channel, dubbed as the "escape–competition" effect (Aghion et al., 2001), introduces an additional force through which competition can foster economic growth and, thus, provides greater flexibility to the model when confronted with micro–level evidence (Aghion et al., 2005). Without doubt, these novel features of the step-by-step innovation framework opened up various new avenues in the analysis of market competition, firm dynamics, and economic growth.

In this piece, I review two recent applications of the step-by-step innovation framework. First, I discuss how this model sheds light on the heated debate about increased market concentration and slowing business dynamism that has been ailing the U.S. economy in the past several decades (Akcigit and Ates, 2019, 2021). Following Akcigit and Ates (2021), this section presents a basic setup, which also helps highlight the key mechanisms of the step-by-step innovation framework. Importantly, the analysis focuses on a specific channel, namely the *knowledge diffusion*, and shows that a decline in this margin is able to go a long way in replicating some prominent trends in the data. The key mechanism that underlies these results lies in the combination of endogenous responses of firms to a decline in *knowledge diffusion* (incentive effect) and and the ensuing shift in the sectoral composition of the economy (composition effect). I also discuss how an extended version of the model á la Akcigit and Ates (2019) can capture a number of other significant aspects of slowing business dynamism.

Next, I discuss how a rare application of this framework in an open economy setting contributes to our understanding of industrial policy and its growth and welfare implications in the face of international technological competition. In this setting, market competition gains an

similar to the entrant innovations in Aghion and Howitt (1992). However, this undirected nature of innovations prevents firms from competing against rivals in their respective sectors. In this respect, the Klette and Kortum (2004) framework, like the Aghion and Howitt (1992) model, does not capture the strategic interaction between rival incumbent firms and, therefore, is not a model of firm competition per se.

international aspect, and firms' innovation incentives peak when faced with competitors having similar technological capacities—i.e., when foreign competition stiffens. Ergo, the growth and welfare effects of industrial or trade policies crucially depend on how these policies alter the strength of foreign competition that domestic firms face, and, consequently, on the dynamic gains from trade they create. I recount some prominent policy implications that follow from this mechanism in light of the analysis by Akcigit et al. (2021).

Other applications of the step-by-step innovation framework include, first and foremost, the seminal work by Aghion et al. (2005). In this paper, the authors document an inverted-U relationship between competition and innovation using data on British industries. The step-by-step innovation theory helps rationalize this inverted-U relationship with the two opposing forces it admits through which competition can affect firms' incentive to innovate: the negative Schumpeterian effect vs. the positive escape–competition effect. The former is the dominating force when the initial level of competition is weak—i.e., an increase in competition induces firms to innovate more—whereas the latter effect dominates when competition is fierce to begin with.[4] Acemoglu and Akcigit (2012) employ this framework to examine the interaction between intellectual property rights (IPR) policies and competition. In particular, they focus on state-dependent policies and find that the optimal policy favors incumbents with a large lead over their rivals, providing them with greater IPR protection. The reason is that this asymmetric policy creates a trickle-down effect, increasing the incremental gain from innovating for leaders with smaller technology advantage over their rivals and thereby boosting their innovation efforts. More recent examples include Akcigit et al. (2018) and Akcigit and Ates (2019, 2021), which we review in this piece. Liu et al. (2019) uses the step-by-step framework to explore the effect of falling interest rates on competition, while Chikis et al. (2021) investigate the robustness of this relationship in an extended version of the model. Another recent example that focuses on the recent dynamics in market concentration and competition is Olmstead-Rumsey (2020). In contemporaneous work on this topic, Cavenaile et al. (2021b) generalize the step-by-step innovation framework to Cournot equilibrium at the product level, allowing for oligopolistic competition between rival firms. Cavenaile et al. (2021a) and Cavenaile et al. (2021c) apply similar frameworks to analyze the growth and welfare effects of advertisement activity and mergers and acquisitions, respectively.

The remainder of the paper is structured as follows. Section 2 discusses some lessons from the step-by-step innovation theory on slowing business dynamism. Section 3 explores the framework in an open–economy setting. Finally, Section 4 concludes.

[4]This result follows from a composition effect that the initial level of competition exerts on the distribution of sectors across technology gaps. For instance, if the product market competition is initially low even when firms have similar technologies—e.g., if firms can collude to share the rents—then, firms' incentive to innovate and escape competition is also low, implying that the share of sectors with "neck-and-neck" firms is high to begin with. In this case, an increase in competition pushes up overall innovation as escape competition effect starts to kick in more forcefully in those neck-and-neck sectors.

2 A Basic Model to Study Slowing Business Dynamism

An extensive set of empirical regularities suggests that business dynamism and market competition have been slowing down in the United States since the early 1980s.[5] By design, step-by-step innovation framework provides an excellent ground to investigate these changes, as it speaks to the nature of competition between firms and the resulting firm dynamics. In this section, we discuss how a basic setup based of step-by-step innovation can help rationalize several prominent symptoms of slowing business dynamism.[6]

A number of crucial features of the model are worth emphasizing: (i) Firms have strategic investment decisions—a key to understanding declining business dynamism, (ii) productivity enhancing innovation decisions are endogenous, (iii) thus, markups are endogenous, depending on the technology gap between competitors, and (iv) a reduced-form parameter governs the process of *knowledge diffusion*. This final feature reflects the exogenous flow of knowledge from the frontier firms to the followers, which allows the follower to close the productivity gap with the leader, bringing them to a neck-and-neck position. The rate of this knowledge diffusion will be crucial in our analysis; in particular, we will show that a weakening in this margin can generate some of the observed changes in the economy.

2.1 Fundamentals

The model economy is in continuous time. Final–good firms produce the final output Y_t in a perfectly competitive market according to the following production technology:

$$\ln Y_t = \int_0^1 \ln y_{jt} \, dj, \tag{1}$$

where y_{jt} denotes the amount of intermediate variety $j \in [0,1]$ used at time t. The final good provides the resource for consumption and R&D investment.

Turning to the intermediate–good production, in each product line j, there are two incumbent firms $i \in \{1,2\}$ that can produce a perfectly substitutable variety of good j. Total output of variety j is given by

$$y_{jt} = y_{ijt} + y_{-ijt},$$

where $-i$ denotes the competitor of firm i, such that $-i \in \{1,2\}$ and $-i \neq i$. Each firm produces using labor l_{ijt} according to the following linear production technology:

$$y_{ijt} = q_{ijt} l_{ijt}.$$

[5] This slowdown is a crucial shift, because a healthy degree of business dynamism in an economy—the perpetual process of entry, growth, downsizing, and exiting of firms—ensures the reallocation of factors toward more productive units, which is in turn the key source of aggregate productivity, and thus, long-run economic growth (Foster et al., 2000).

[6] This analysis follows the steps of Akcigit and Ates (2021).

Here, q_{ijt} denotes the labor productivity of firm i. These firms compete for market leadership à la Bertrand. Higher labor productivity gives a firm a cost advantage over its rival, allowing it to supply good j to the entire market. Accordingly, firm i is the market leader and $-i$ is the follower in j if $q_i > q_{-i}$. The two firms are neck-and-neck if $q_i = q_{-i}$.

Firms invest in innovative activity to improve their productivity. An innovation increases the innovating firm's productivity level proportionally by a factor $\lambda > 1$ such that $q_{ij(t+\Delta t)} = \lambda q_{ijt}$. We set the initial value $q_{ij0} = 1$. Then, the productivity levels at time t becomes $q_{ijt} = \lambda^{n_{ijt}}$, where n_{ijt} captures the number of productivity improvements that took place by firm i since time 0. Thus, the productivity difference between two firms reflects the difference between the total number of rungs these firms' production technologies build on. The productivity level of a firm relative to its rival is given by

$$\frac{q_{ijt}}{q_{-ijt}} = \frac{\lambda^{n_{ijt}}}{\lambda^{n_{-ijt}}} = \lambda^{n_{ijt}-n_{-ijt}} \equiv \lambda^{m_{ijt}},$$

where $m_{ijt} \in \{-1, 0, 1\}$ defines the technology gap between the firm i and $-i$ in sector j. For simplicity, we set the maximum size of this difference to one; therefore, the economy admits two types of product lines: leveled and unleveled. The technology gap between firms in a product line is a sufficient statistic to describe firm-specific payoffs. Hence, we will drop industry subscript j and use the notation $m_{it} \in \{-1, 0, 1\}$ whenever m is specified to denote a firm-specific value. Likewise, we will use $m_{jt} \in \{0, 1\}$ to index sectors that are leveled or unleveled.

Firms choose the arrival rate of an innovation x_{ijt}. We take the associated cost of generating x_{ijt} to be of quadratic shape such that

$$R_{ijt} = \alpha \frac{x_{ijt}^2}{2} Y_t,$$

with R_{ijt} denoting the R&D spending. The scale parameter is given by α, and we assume that the cost scales with aggregate output.

An additional feature of the model is that knowledge diffuses from the leader to the follower at an exogenous Poisson flow rate δ. Knowledge diffusion enables the follower to catch up with the leader's productivity level, bringing both firms to a neck-and-neck position. In this sense, δ is a reduced–form representation of spillovers from the leaders to the followers: It captures the mechanisms through which technologically laggard firms learn from products and processes at the technology frontier (Bloom et al., 2013).[7] Notice that this margin is different than the endogenous R&D decisions of firms in that it occurs exogenously and independent of firms' R&D investments.[8]

[7] An alternative interpretation is that this margin captures *intellectual property rights (IPR) protection* (see Acemoglu and Akcigit, 2012). A leader's patent expires with the flow rate δ, allowing the follower to replicate the frontier technology and catch-up with the leader. Then, a lower value of δ implies higher protection and lower catch-up.

[8] This margin also helps the economy maintain a competitive environment at the industry level, preventing firms falling too far behind the technology frontier. This effect is more evident in a richer setting where the technology gap

Closing the model, a unit measure of representative households consume the final good admitting log-utility preferences and supply a unit of labor inelastically at the competitive wage rate w_t. They also own the firms in the economy and earn interest on their assets at rate r_t.

2.2 Balanced Growth Path

In the remainder of our theoretical analysis, we focus on a balanced growth path equilibrium. The equilibrium is Markov perfect, with strategies depending only on the payoff-relevant state variable $m \in \{-1, 0, 1\}$, and on BGP, all aggregate variables grow at the same rate g. In this section, we will list the objects that are most relevant to our analysis and defer the full set of equilibrium relationships to Appendix A. Henceforth, we will drop the indices i, j and t unless it causes confusion.

The optimization of the representative final good producer results in a unit-elastic demand for good $j \in [0, 1]$:

$$y_{ij} p_{ij} = Y, \tag{2}$$

where p_{ij} is the price of intermediate j charged by the producing monopolist i. Under the linearity of the production function, an intermediate producer's marginal cost is

$$MC_{ij} = \frac{w}{q_{ij}}. \tag{3}$$

The marginal cost of production increases in the unit labor cost w and decreases in labor productivity q_{ij}. Faced with a unit-elastic demand from the final–good producer and engaging in price competition with its rival, the intermediate producer follows a limit pricing rule in equilibrium. More precisely, it sets its price to the marginal cost of the competitor:

$$p_{ij} = \frac{w}{q_{-ij}}. \tag{4}$$

Consequently, the equilibrium quantity of intermediate good j is simply

$$y_{ij} = \frac{q_{-ij}}{\omega} \quad \text{for} \quad q_{ij} \geq q_{-ij} \tag{5}$$

and $y_{ij} = 0$ otherwise.[9] Here, ω is the normalized wage rate in the economy is defined as $\omega \equiv \frac{w}{Y}$. It also measures the aggregate labor share of output.

The operating profits of an intermediate firm exclusive of its R&D expenditures is

$$\pi(m_i) = \begin{cases} \left(1 - \frac{1}{\lambda}\right) Y & \text{if} \quad m_i = 1 \\ 0 \quad \text{if} & m_i \in \{0, -1\} \end{cases}$$

between firms can grow beyond one step (see Akcigit and Ates, 2019).

[9] We assume that production in an industry is randomly distributed when firms are neck-and-neck.

Similarly, the markups in leveled ($m_j = 0$) and unleveled ($m_j = 1$) sectors are given as

$$Markup_j = \frac{p_{ij}}{MC_{ij}} - 1 = \begin{cases} \lambda - 1 & \text{if} \quad m_j = 1 \\ 0 & \text{if} \quad m_j = 0 \end{cases}$$

Notice that only leaders can charge a positive level of markup and profit. Therefore, aggregate levels of profits and markups are determined by the distribution of intermediate lines across leveled and unleveled ones, whose evolution depends crucially on endogenous innovation decisions of firms. As such, these objects become endogenous to firms' strategic decisions taking each other's responses into account, and therefore, this setting provides a convenient starting point to analyze their dynamics.

We now define two more endogenous objects that prove useful in characterizing key aggregate variables in equilibrium. First, the aggregate productivity index of the economy is given by

$$Q \equiv \exp\left(\int_0^1 \ln q_j dj\right).$$

Next, the share of unleveled industries, which also acts as a proxy for the level of *market concentration*, is denoted by

$$\mu \equiv \int_0^1 \mathbb{I}(q_{ij} \neq q_{-ij}) dj.$$

The aggregate wage rate follows from the final good production function (1) and the equilibrium intermediate goods (5):

$$w = \frac{Q}{\lambda^\mu}. \tag{6}$$

Then, the labor market clearing condition, $\int_0^1 l_{jt}\, dj = 1$, implies the following labor share ω:

$$\omega = 1 - \mu \frac{(\lambda - 1)}{\lambda}. \tag{7}$$

The labor share decreases in the level of market concentration μ and the markup parameter λ. Therefore, the market concentration and the labor share are negatively correlated.

Finally, equations (6) and (7) yields the level of final output as

$$Y = \frac{Q}{\lambda^\mu \left[1 - \mu \frac{(\lambda-1)}{\lambda}\right]}. \tag{8}$$

Notice that, along the BGP, final output scales with the aggregate productivity. Consequently, the long-run growth rate of aggregate productivity also determines the growth rate of output and consumption. Moreover, equation (8) implies that the distribution of markups leads to static efficiency losses. In fact, at the minimum ($\mu = 0$) or maximum ($\mu = 1$) level of concentration we have $Y = Q$. However, additional efficiency losses arise in the economy when markups are unevenly distributed across the sectors.

Firm Values and Innovation We denote the stock market value of a firm that is in state $m_i \in \{-1, 0, 1\}$ by V_{m_i} and define $v_{m_i} \equiv V_{m_i}/Y$. Then, the normalized value function of an incumbent firm that is one-step ahead, i.e., $m_i = 1$, is given by

$$\rho v_1 = \max_{x_1} \left\{ \left(1 - \frac{1}{\lambda}\right) - \frac{x_1^2}{2} + x_1 \left[v_1 - v_1\right] + \left(x_{-1} + \delta\right) \left[v_0 - v_1\right] \right\},$$

where x_{m_i} for $m_i \in \{-1, 0, 1\}$ denotes the innovation rate of a follower, neck-and-neck firm, and a leader, respectively. The first two terms on the right-hand side combined yield the profits net of R&D expenditure. The third term captures the result of a successful innovation by the leader. When the one-step leader innovates, the gap difference does not increase because of the imposition of an upper limit on the potential size of gaps. Therefore, the one-step leader will optimally avoid investing in R&D—i.e., $x_1 = 0$ holds in equilibrium. The fourth term reflects the result of an innovation by the follower or the exogenous knowledge diffusion happening at rate δ. In these cases, the leader loses its productivity advantage and becomes neck-and-neck with the competitor.

Reciprocally, the value of function of a follower is defined as

$$\rho v_{-1} = \max_{x_{-1}} \left\{ -\frac{x_{-1}^2}{2} + \left(x_{-1} + \delta\right) \left[v_0 - v_{-1}\right] \right\}. \tag{9}$$

The follower does not produce and, therefore, does not earn any profits. Yet, the forward–looking firm invests in R&D with the prospect of taking over the leader through successive (step-by-step) innovations. The catch-up can also happen at the exogenous flow rate δ. Finally, the value of a neck-and-neck incumbent is given by

$$\rho v_0 = \max_{x_0} \left\{ -\frac{x_0^2}{2} + x_0 \left[v_1 - v_0\right] + x_0 \left[v_{-1} - v_0\right] \right\}.$$

A successful innovation of the neck-and-neck firm makes it a leader, whereas an innovation by the competitor makes it a follower.

The first order conditions of the problems above yield the following optimal innovation decisions:

$$\begin{aligned} x_1 &= 0 \\ x_0 &= v_1 - v_0 \\ x_{-1} &= v_0 - v_{-1}. \end{aligned} \tag{10}$$

With these values, the BGP value of μ, the share of unleveled sectors, becomes

$$\mu = \frac{2x_0}{2x_0 + x_{-1} + \delta}. \tag{11}$$

Finally, the equilibrium growth rate of this economy is given by

$$g = 2x_0(1-\mu)\ln\lambda. \tag{12}$$

The growth rate of the economy is determined by innovations of neck-and-neck firms.[10] Interestingly, innovations by followers in unleveled sectors do not contribute to the BGP growth, because their innovations do not push the technology frontier forward; rather, they only help the followers catch up with the frontier. Therefore, increased market concentration (μ)—i.e., a higher share of unleveled sectors in the economy—has a negative impact on economic growth (g).[11]

2.3 Impact of Knowledge Diffusion, δ

In this section, we discuss some theoretical predictions of the framework introduced above, with a particular focus on the effect of a decline in the intensity of knowledge diffusion on firms' innovation rates and its distributional consequences. The following lemma forms the basis of the main results.

Lemma 1 *The following results hold in the BGP.*

1. *Neck-and-neck firms have higher innovation intensity than laggard firms.*
2. *An increase in knowledge diffusion decreases innovation efforts. The decline is even more drastic for the neck-and-neck firms.*

Proof. *See Akcigit and Ates (2021)* ■

The first point of Lemma 1 is the well-known escape-competition effect in step-by-step innovation models, namely the intensification of innovation effort by neck-and-neck firms to get ahead of their competitor and avoid competition. The second result arises because the value of being a leader increases disproportionately as the exogenous risk of losing the position declines. These results lead to the following corollary.

[10] Notice that this result is an artifact that we restricted the maximum gap between firms to one. In a richer setting where m can take other positive values, leader firms—except for the leader at the largest gap—would also have incentives to innovate and contribute to the aggregate growth of the economy. To see an example of such a setting, see Akcigit and Ates (2019).

[11] In a more general setting where technology gaps could be larger than one step (see Akcigit and Ates, 2019), leaders in the unleveled sectors (except at the maximum gap) still innovate and contribute to aggregate growth. However, the larger technology lead implies that they are subject to less competitive pressure, hence, their incentives to innovate weaken as the technological lead widens. Therefore, the negative correlation between aggregate market concentration and economic growth holds also in the more general setting.

Corollary 1 *In the BGP, a decrease in knowledge diffusion increases the share of unleveled sectors.*

Proof. *See Akcigit and Ates (2021).* ■

Corollary 1 contrasts two BGP equilibria with different knowledge diffusion rates. As the diffusion intensity decreases, the rate of innovation by neck-and-neck firms rises in response more than that by followers, leading to a subsequent increase in the measure of unleveled sectors. This compositional shift is key to the results discussed next.

2.4 Reduction in Knowledge Diffusion and Business Dynamism

Next, we discuss the predictions of the model with regard to some of the symptoms that characterize the slowing business dynamism in the U.S. economy.

Prediction 1 *Market concentration rises in response to lower knowledge diffusion.*

Market concentration has been on the rise in the U.S. economy, as documented by Autor et al. (2017a,b) and Grullon et al. (2017). The model generates a similar prediction in response to lower knowledge diffusion. Notice that in the model, markups and profits vanish in leveled industries because of limit pricing, reflecting the intense competition when firms are neck-and-neck. Moreover, sales are equalized. As a result, the aggregate Herfindahl-Hirschman index (HHI) reads as

$$\begin{aligned} HHI &= \mu \times [(100\%)^2 + (0\%)^2] + (1-\mu) \times [(50\%)^2 + (50\%)^2] \\ &= 0.5 + 0.5\mu. \end{aligned}$$

The HHI, the key measure of market concentration, increases in the measure of unleveled industries (μ), whose BGP value is given by equation (11) and decreases with knowledge diffusion (Corollary 1). Two channels drive this result. First, a lower δ means a smaller frequency at which followers learn from the leaders; hence, pushing the distribution toward unleveled sectors. Second, reduced knowledge diffusion increases the return to being the market leader, which incentivizes neck-and-neck firms to innovate relatively more, as they are much closer to capturing the market than a follower. This relatively stronger increase in innovation by neck-and-neck firms also expands the share of unleveled industries. Hence, the market concentration increases with lower knowledge diffusion, i.e.,

$$\frac{d(HHI)}{d\delta} < 0.$$

Prediction 2 *Markups rise in response to lower knowledge diffusion.*

Lower knowledge diffusion helps the model replicate the rise in markups, a feature of the U.S. economy as documented by a set of recent papers (see Nekarda and Ramey, 2013; De Loecker et al., 2017; Gutiérrez and Philippon, 2017; Eggertsson et al., 2018; Hall, 2018, among others). In the model, only leaders in unleveled sectors can set positive markups. Therefore, the average markup in this economy is

$$\begin{aligned} Average\_markup &= \mu \times (\lambda - 1) + (1 - \mu) \times 0 \\ &= \mu \times (\lambda - 1). \end{aligned}$$

The average markup is proportional to μ, and given corollary 1, it increases when knowledge diffusion decreases, i.e.,

$$\frac{d(Average\_markup)}{d\delta} < 0.$$

Prediction 3 *Profits rise in response to lower knowledge diffusion.*

Similar to markups, the profit share in the U.S. economy has been increasing as well. The model generates a consistent prediction, with operating profits as a share of GDP rising in response to lower diffusion. The aggregate profit share is simply

$$Profit/GDP = \mu \times \left(1 - \frac{1}{\lambda}\right). \tag{13}$$

As in the preceding analysis, a reduction in knowledge diffusion also causes a rise in she ratio of operating profits to GDP:

$$\frac{d(Profit/GDP)}{d\delta} < 0.$$

Prediction 4 *The labor share of output declines in response to lower knowledge diffusion.*

Another prediction of the model consistent with recent trends in the U.S. economy is the decline in the labor share of output (see Karabarbounis and Neiman, 2013; Elsby et al., 2013; Lawrence, 2015 for the empirical evidence). The labor share in the above economy is

$$\begin{aligned} Labor\_share &= (1 - \mu) \times 1 + \mu \times \frac{1}{\lambda} \\ &= 1 - \mu \times \left(1 - \frac{1}{\lambda}\right). \end{aligned} \tag{14}$$

Notice that the labor share and markups (profits) go in opposite directions. Labor is the only input for intermediate production, and when business owners generate some additional windfall as a fraction of the output, it comes at the expense of reduced labor compensation. Consequently, the labor share decreases with lower diffusion rate:

$$\frac{d(Labor\_share)}{d\delta} > 0.$$

Prediction 5 *The productivity gap between leaders and followers widens with lower knowledge diffusion.*

Consistent with the evidence in Andrews et al. (2015, 2016), the model generates a widening productivity gap between leaders and followers as knowledge diffusion weakens. The productivity of the market leader relative to the follower is 1 in leveled industries and λ in unleveled industries. Therefore, the average relative productivity can be expressed as

$$\begin{aligned} Average\_productivity\_gap &= \mu \times \lambda + (1 - \mu) \\ &= 1 + \mu \times (\lambda - 1). \end{aligned}$$

Together with Corollary 1, this expression implies that when the average productivity gap between the leaders and followers widen when knowledge diffusion slows. Therefore,

$$\frac{d(Average\_productivity\_gap)}{d\delta} < 0.$$

2.5 Discussion of Further Implications

Akcigit and Ates (2021) review a number of additional characteristics of slowing business dynamism and declining competition in the United States, and the model could well speak to those aspects, too. First, some recent work documents a negative relationship between market concentration and the labor share in an industry Autor et al. (2017b), Barkai (2017), and Eggertsson et al. (2018). Similarly, the labor share is the largest in level industries in the model—i.e., when concentration is the lowest—as neck-and-neck firms do not generate profits. When a level industry becomes unleveled, market concentration rises and the labor share decreases—the leader retains part of its revenue in profits. As such, the model generates the empirical negative association between market concentration and the labor share.

A well-documented observation associated with slowing business dynamism in the United States is the decline in firm entry rate and the share of young firms in economic activity (Decker et al., 2016; Karahan et al., 2016; Gourio et al., 2014). While the simple model is silent on these closely tied observations, we can already develop some intuition on the implications of free entry in this framework. Suppose that we introduce firm entry to the model, with entrants replacing either followers ($m_i = -1$) with probability μ or neck-and-neck firms ($m_i = 0$) with probability $1 - \mu$. This specification would be consistent with the well-established empirical evidence that new firms start small and some manage to grow over time. In this setting, a decline in knowledge diffusion would push up market concentration. Consequently, the probability that an entrant competes against a dominant market leader ($m_i = 1$) increases, which would discourage new firm creation, as entrants are forward–looking agents. In addition, the direct effect of lower δ and the indirect effect of lower firm entry imply that the economic activity by young firms decreases as well.

Two other trends regarding the average growth rate of incumbents indicate a decline in business dynamism concern: (i) Job reallocation has slowed, all the while (ii) the dispersion of firm growth has decreased (Decker et al., 2016). The model also speaks to the evolution of these variables, but the response of these variables to a decline in knowledge diffusion is ambiguous. Note that two forces drive the changes in firm growth rates: (i) the composition of industries (μ) and (ii) the innovation incentives in each of those industries. First, both followers and neck-and-neck firms increase their investment to innovate in response to lower δ since the value of market leadership increases, and these firms are forward–looking agents. This is the *positive incentive effect*. Yet, a reduction in knowledge diffusion means the share of unleveled sectors—sectors where aggregate investment in innovation is lower—rises. This is the *negative composition effect*. Therefore, the net effect of these forces on firm growth and job reallocation is ambiguous and depends on their relative magnitudes.

As the preceding discussion indicates, the theoretical analysis produces ambiguous implications for some relationships, and a quantitative analysis is necessary to gain further insights. Indeed, Akcigit and Ates (2019) build a version of the model described here that is versatile enough to conduct a quantitative analysis of the aforementioned empirical trends. First and foremost, it extends the basic model by adding the entry margin. As such, the extended model is where Aghion and Howitt (1992) meets Aghion et al. (2005): creative destruction and firm turnover due to firm entry as in Aghion and Howitt (1992) complements the strategic interaction aspect of the step-by-step innovation framework. Second, they extend the analysis to transitional dynamics in order to replicate the experience of the U.S. economy in the past several decades. In addition, the quantitative framework allows the discussion of other potential channels that could have contributed to the observed trends in the data and the assessment of their relative importance. In their analysis, the authors find that the strongest driver of slower business dynamism in the United States has been a decline in the intensity of knowledge diffusion from the frontier firms to laggard ones.

Reduction in knowledge diffusion is able to account for these trends as follows. When knowledge diffusion slows down over time, as a direct effect, market leaders are shielded from being copied, which helps them establish stronger market power. When market leaders have a bigger lead over their rivals, the followers get discouraged; hence, they slow down. The productivity gap between leaders and followers opens up. The first implication of this widening is that market composition shifts to more concentrated sectors. Second, limit pricing allows stronger leaders (leaders further ahead) to charge higher markups, which also increases the profit share and decreases the labor share of gross domestic product (GDP). Since entrants are forward looking, they observe the strengthening of incumbents and get discouraged; therefore, entry goes down. Discouraged followers and entrants lower the competitive pressure on the market leader: When they face less threat, market leaders relax and they experiment less. Hence, overall dynamism and experimentation decrease in the economy. Consequently, with lower innovation investment, productivity growth slows over time, causing the equilibrium interest rate to fall. As such, the model provides an endogenous mechanism for declining interest rates over time—a widely discussed phenomenon in the United States (Summers, 2014).

3 International Trade and Competition

The step-by-step innovation framework also helps shed light on the nature of international technological competition and how competition and innovation interact in open economies. For instance, recent empirical studies find diverging results when analyzing the effect of Chinese import penetration on domestic innovation outcomes. Similarly, a large literature explores the effect of trade liberalization on innovation and growth outcomes (Shu and Steinwender, 2019). Inherently, the step-by-step innovation model provides an immensely suitable theory to understand the economic mechanisms behind these relationships: Unlike the first-generation Schumpeterian models, it can capture not only the negative effects of competition on innovation, but also potential positive effects via the escape–competition channel. We next discuss a recent application of this framework in an open economy setting.

3.1 Basic Setup

We consider the following world economy in continuous time that consists of two large open countries. In each country, representative final–good producers operate in perfectly competitive markets and produce the final output combining domestic fixed factor F_c (supplied inelastically) and a measure of intermediate varieties via the following technology:

$$Y_{ct} = \frac{F_c^{\beta}}{1-\beta} \int_0^1 \left(q_{Ajt}^{\frac{\beta}{1-\beta}} k_{Ajt} + q_{Bjt}^{\frac{\beta}{1-\beta}} k_{Bjt} \right)^{1-\beta} dj,$$

with the parameter β denoting the inverse of the elasticity of substitution across varieties j as well as the share of fixed factor. In this expression, k_{ijt} is the amount of variety j, and q_{ijt} is the associated quality. Note that in this version of the model, intermediate–good firms differ in their product quality, whereas in the previous section, the heterogeneity lied in labor productivity. As we shall see, this structure together with a Cobb-Douglas production function generates isoelastic—as opposed to unit-elastic—demand for intermediate varieties, which, in turn, leads to firm–level profits to scale in product quality, providing for any intermediate–good firm an incentive to innovate.[12]

Intermediate goods can potentially be produced by infinitely-lived incumbent firms from both countries using a linear round-about production technology with a constant marginal cost η. Importantly, the intermediate varieties are tradable across countries, and for simplicity, we assume away any trade costs.[13] In each product line, one firm from each country engages in Bertrand price competition to capture the market. As we shall see, the firm with a higher product quality will have an edge in this competitive environment.

[12]Recall that in the previous section, the leader firm optimally chooses to not undertake R&D investment.

[13]The final good is also assumed to be tradable; however, this assumption does not play a role for the dynamics of the model. Rather, it helps ensure trade balance in both economies in a simple and innocuous manner.

The nature of technological competition and innovations is analogous to the framework in the previous section, except that they pertain to product quality instead of labor productivity.[14] To avoid repetition, we keep their description brief unless otherwise is necessary. Again, the firm with a better technology—product quality, in this case—is the market leader, i.e., firm i in line j is the leader iff $q_{ijt} > q_{-ijt}$. Follower and neck-and-neck stages are defined accordingly. Firms can invest in R&D to improve their product quality, with each innovation improving the product quality a multiplicative factor λ. The variable m_{ijt} captures again the technology gap between firm i and $-i$ in line j, reflecting their relative quality levels. We again restrict the maximum gap to be one step.

Setting up the static equilibrium, the final–good production function and the absence of trade costs imply that the final good producers in both countries purchase each variety from the supplier with the higher quality. The final good producers' optimization generates the following (inverse) demand schedule for good j:[15]

$$p_j = F_c^{\beta} q_j^{\beta} k_j^{\beta}. \tag{15}$$

Faced with this demand, the profit maximization problem of the monopolist leader reads as

$$\max_{k_j} \left(p_j\left(k_j\right) - \eta\right) k_j \quad \text{subject to equation (15).}$$

The optimal production and price decisions are[16]

$$p_j = \frac{\eta}{1-\beta} \quad \text{and} \quad k_j = \left[\frac{\eta}{1-\beta}\right]^{-\frac{1}{\beta}} F q_j.$$

Notice that these apply to both domestic sales and exported goods as there are no trade costs. Consequently, the profits of a leading firm is given by

$$\pi_j = \Xi q_j,$$

with $\Xi \equiv \left[\frac{\beta-1}{\beta}\right]^{-\frac{1}{\beta}} \beta F$. Firms that are not in a leading position do not earn any positive profits.

In this setting, the dynamic problems of forward-looking firms are summarized by the fol-

[14] Still, the implications of this setup differ substantially from those in the previous model. For instance, it generates additional incentives for innovation: even the market leader try to innovate, as equilibrium profits are linear in product quality. For the details of this general setup, please see Akcigit et al. (2021).

[15] We drop the country subscript, because the nation of origin does not matter for the final–good producer. We also remove the time subscript unless it causes confusion.

[16] For the sake of clarity, we maintain the assumption that incumbents play a two-stage game à la Acemoglu et al. (2012), which ensures that the leader can charge the unconstrained monopoly price. Before setting optimal prices in the second stage, firms need to pay an infinitesimally small amount in the first stage. By backward induction, only the firm with the quality advantage enters the game.

lowing value functions:

$$r_{At}V_{A1t}\left(q_{jt}\right)-\dot{V}_{A1t}\left(q_{jt}\right)=\max_{x_{A1t}}\left\{\begin{array}{c}2\Xi q_{jt}-\chi x_{A1t}^{2}q_{jt}+x_{A1t}\left[V_{A1t}\left(\lambda q_{jt}\right)-V_{A1t}\left(q_{jt}\right)\right]\\-x_{B-1t}\left[V_{A0t}\left(q_{jt}\right)-V_{A1t}\left(q_{jt}\right)\right]\end{array}\right\} \tag{P1}$$

$$r_{At}V_{A0t}\left(q_{jt}\right)-\dot{V}_{A0t}\left(q_{jt}\right)=\max_{x_{A0t}}\left\{\begin{array}{c}-\chi x_{A0t}^{2}q_{jt}+x_{A0t}\left[V_{A1t}\left(\lambda q_{jt}\right)-V_{A0t}\left(q_{jt}\right)\right]\\-x_{B0t}\left[V_{A-1t}\left(q_{jt}\right)-V_{A0t}\left(q_{jt}\right)\right]\end{array}\right\} \tag{P2}$$

$$r_{At}V_{A-1t}\left(q_{jt}\right)-\dot{V}_{A-1t}\left(q_{jt}\right)=\max_{x_{A-1t}}\left\{\begin{array}{c}-\chi x_{A-1t}^{2}q_{jt}+x_{A-1t}\left[V_{A0t}\left(\lambda q_{jt}\right)-V_{A-1t}\left(q_{jt}\right)\right]\\-x_{B1t}\left[V_{A-1t}\left(\lambda q_{jt}\right)-V_{A-1t}\left(q_{jt}\right)\right]\end{array}\right\} \tag{P3}$$

Equations (P1)-(P3) describe the generic problem of a leader, a neck-and-neck firm, and a follower, respectively. Only the leader earns profits and it does so selling to both countries.[17] All firms face a quadratic R&D cost, with x_{ijt} denotes the Poisson flow rate of innovation. When the leader innovates, it improves its quality level, but its position does not change. However, if its rival innovates they become neck-and-neck. In a neck-and-neck sector, an innovation helps the neck-and-neck firm gain a lead over its rival, but if the competitor innovates, the neck-and-neck firm becomes a follower. Finally, an innovation helps the follower catch up with the leader. In case of leader's innovation, the gap remains the same, reflecting spillovers from the frontier firm to the follower, helping the follower remain in the competitive race.

Next, we highlight the forces that international technological competition exerts on incumbents' innovation incentives.

3.2 Dynamic Effects of Openness and Escape–Competition Effect

In order to emphasize the dynamic strategic interaction between intermediate producers introduced by foreign competition, we focus on a special case of our model with minimal incentives for quality improvements. This assumption does not eliminate all incentives for innovation. Rather, it dampens a particular motive—improving the product quality—to isolate motives that are generated by the competitive race—improving the position relative to the rival. Precisely, we assume that $\lambda = 1 + \varepsilon$, where ε is arbitrarily close to zero, implying that quality improvements from innovations are infinitesimal. Consequently, innovation incentives are driven only by discouragement and escape-competition effects. For convenience, we also take $\chi = 1/2$ and confine our analysis to the BGP. The following proposition argues that, in this environment, firms in neck-and-neck position have the highest innovation intensity.

Proposition 1 *The above assumptions imply that*

1. *the innovation intensity becomes the highest at neck-and-neck position;*
2. *the followers innovate at the same intensity and strictly less than the neck-and-neck firms;*

[17] Bertrand competition in neck-and-neck sectors pushes their rents down to zero.

3. *the leaders do not innovate.*

Formally, $x_0 > x_{-1} > x_1 = 0$.

Proof. See Appendix B. ■

Proposition 1 formalizes the fact that the positive effect of foreign competitive pressures on innovation incentives becomes the strongest when firms compete against rivals producing goods of similar quality. This effect is analogous to the one in closed-economy step-by-step models—namely, the "*escape-competition*" effect—but it gains an international aspect in the context of a small open economy. As such, this effect allows the model to capture potential boost from international competition to firms' innovative activity. To be sure, the model still reflects the potential downside of foreign competition: domestic firms that fall behind the competitive race get discouraged in their innovative effort (*discouragement effect*). Hence, this versatility of the step-by-step innovation framework makes it a very convenient and realistic setting to study the effect of foreign competition on economic growth and analyze industrial policy in open economies. In Akcigit et al. (2021), we do so using a more advanced version of the model, and several interesting implications arise.[18]

First, Akcigit et al. (2021) studies a more general setting that features trade costs and allows the productivity gap to open up more than one step. In such a setting, the existence of trade costs modifies the nature of escape–competition effect in significant ways. Raising the cost of imports, they help some domestic firms retain their domestic market, even though their foreign competitor has a better quality. Conversely, it is not enough for a domestic firm to have superior product quality to export; the quality advantage should be large enough to compensate for the additional costs. Accordingly, the "neck-and-neckness" arises at two different stages of international competition, and the innovation effort of a firm intensifies at two different points, for two similar yet distinct reasons: to protect domestic profits when the firm is a technological laggard; and to access export markets when the firm is a technology leader. As a result, the innovation effort schedule of a firm as a function of the quality gap relative to its rival has double-peaked shape, and empirical evidence provided corroborates this prediction. Notice the difference with respect to the standard model: in the standard setting, competition and innovation effort (escape–competition effect) peaks only when the two firms have exactly the same quality, translating to a single-peaked innovation effort schedule. This heterogeneity has subtle implications for policy, as discussed in detail by Akcigit et al. (2021).

The quantitative investigation of this model uncovers intriguing policy implications. First, increased foreign competition reduces the need for R&D subsidies to prop up domestic innovation. In particular, as bilateral trade costs decline—i.e., the world becoming more open—the

[18] For excellent surveys of empirical studies on the nexus of foreign competition, innovation, and economic growth, please see the recent reviews by Akcigit and Melitz (2021), Melitz and Redding (2021), which is in this volume, and Shu and Steinwender (2019).

optimal level of R&D subsidy that a policymaker would set for a given horizon decreases as well. This result also hinges crucially on the escape–competition effect. Lower trade costs imply that more domestic firms face stronger competitive threat, inducing them to innovate more intensively. Given the technology gap in a sector, these stronger incentives push most firms closer to the optimal innovation effort, reducing the need for R&D subsidies to correct for deficient domestic innovative activity.

Another key policy implication is that it is optimal for the policymaker to slash trade barriers to zero—at all policy horizons and even unilaterally. The optimality of removing unilateral trade barriers unilaterally is a novel finding in the international trade literature and hinges crucially on the effect of protectionist policies on market competition and innovation incentives.[19] While trade barriers protect some firms in the short run, helping them retain production and profits, this protection from competitive pressure that foreign rivals could exert reduces the incentives of domestic firms to innovate and improve the competitiveness of their products. Notice that this effect emerges on top of the fact that trade barriers deprive the economy of some superior foreign products through distortions they create in relative prices. This negative dynamic effect of trade barriers on innovation incentives translates into lower productivity growth in the economy over time and becomes the dominant margin in welfare calculations of the policymaker. Therefore, the policymaker chooses optimally to curtail trade barriers even unilaterally.

Finally, the rich dynamics of this model propose a rationale that can reconcile the seemingly contradictory findings of empirical studies that evaluate the effect of import penetration on domestic innovative activity.[20] As discussed, the model implies that stiffer competition encourages some domestic firms to increase their innovation effort and discourages some others, depending on their position in the competitive race. Then, the overall effect of import penetration on domestic innovation outcome would depend on the composition of sectors in an economy, i.e., whether most domestic firms are competitive enough relative to their foreign rivals to begin with. Indeed, Akcigit et al. (2021) corroborate this intuition, showing numerically that overall innovation effort in an economy can decrease or increase in response to lower barriers to import. Again, the ultimate response is determined by the initial sectoral composition in terms of competitiveness of domestic firms.

[19]This effect is dominant especially in longer horizons. In the short term, the positive effect of higher trade barriers on domestic wages, which in turn reduces the competitiveness of an economy, is an additional margin that makes positive trade barriers suboptimal. However, the analysis of another version of the model that lacks this margin demonstrates that for relatively far-sighted policymakers, a unilateral removal of trade barriers is still the optimal policy (see Akcigit et al., 2018).

[20]Bloom et al. (2016) argue that Chinese import competition induced innovative activity in exposed domestic sectors in 12 European countries they analyze (see also Coelli et al., 2016; Gorodnichenko et al., 2010; Iacovone et al., 2011; and Iacovone, 2012), whereas Autor et al. (2020) find a negative effect on U.S.firms and sectors (see also Hashmi, 2013 and Hombert and Matray, 2015). Yet some other papers including Aghion et al. (2017) find ambiguous results.

4 Conclusion

Schumpeterian growth theory, with its intuitive and tractable modeling of incentives for innovation, has envisaged long-run productivity growth as an endogenous outcome and fundamentally altered economists' thinking about economic growth. A key margin that affects firms' innovation incentives is the degree of competition among them. The step-by-step innovation framework zeroes in on this margin, and thanks to its realistic yet tractable account of competition and innovation, it has a unique place in the endogenous growth tradition. Distinctively, incumbent firms respond strategically to each other via endogenous innovation decisions and depending on the magnitude of competitive pressure. The fundamental insights this theory offers—importantly, the escape–competition effect, among others—widened the relevance of Schumpeterian growth models greatly and enhanced its applicability to new lines of research. In this piece, I reviewed two such applications.

In one direction, the step-by-step innovation model sheds light on the causes of slowing business dynamism in the United States (Akcigit and Ates, 2019, 2021). A particular mechanism—a decline in knowledge diffusion—seems to have plagued the U.S. economy: through the lens of the model, this decline distorts the competition between market leaders and followers, rigging the game in favor of the former, and leads ultimately to lower innovation incentives for firms in both groups. In another line of research, the step-by-step innovation model enhances our understanding of trade and industrial policies in open economies. Among others, it helps expose the distortionary dynamic effect of protectionist policies, which result in novel policy implications. The key mechanism again emphasizes the significance of competitive pressures—in this case, exerted by foreign rivals—in driving innovative activity by domestic firms.

The two examples highlighted in this piece illustrate the broad applicability and policy relevance of the step-by-step innovation framework. Yet a casual examination indicates that this framework has not been utilized to its fullest potential. I contend that its outreach will quickly expand in the years to come, and, especially with the latest computational techniques and computing power, its quantitative applications will advance further to help us address many other interesting questions at the juncture of competition, innovation, and economic policy.

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Appendices

A Equilibrium

We focus on the Balanced Growth Path (BGP) Markov perfect equilibrium, with equilibrium strategies depending only on the payoff-relevant state variable $m \in \{-1, 0, 1\}$ and all aggregate variables growing at the same rate g while firms' innovation rates remain constant. Henceforth, we will drop the indices i, j and t when it causes no confusion and use only the pay-off relevant state variable m.

1. Equilibrium interest rate:
$$r = g + \rho, \tag{A.1}$$
where g is the BGP growth rate of consumption.

2. Demand schedule for the intermediate good $j \in [0, 1]$:
$$y_{ij} = \frac{Y}{p_{ij}}, \tag{A.2}$$
where p_{ij} is the price of intermediate j charged by the producing monopolist i.

3. Intermediate producer's marginal cost:
$$MC_{ij} = \frac{w}{q_{ij}} \tag{A.3}$$
with w denoting the wage level.

4. Equilibrium intermediate good quantities:
$$y_{ij} = \frac{q_{-ij}}{\omega} \quad \text{for} \quad q_{ij} \geq q_{-ij} \tag{A.4}$$
and $y_{ij} = 0$ otherwise, with the normalized aggregate wage rate given as $\omega \equiv w/Y$.

5. Optimal production employment of the intermediate producer:
$$l_i = \frac{y_i}{q_i} = \frac{1}{\omega \lambda^{m_i}} \quad \text{for} \quad m_i \in \{0, 1\}. \tag{A.5}$$

6. Operating profits of an intermediate firm (exclusive of its R&D expenditures):
$$\pi(m_i) = \begin{cases} \left(1 - \frac{1}{\lambda}\right) Y & \text{if} \quad m_i = 1 \\ 0 \quad \text{if} & m_i \in \{0, -1\} \end{cases}$$

7. Markups in leveled ($m_j = 0$) and unleveled ($m_j = 1$) sectors:

$$Markup_j = \frac{p_{ij}}{MC_{ij}} - 1 = \begin{cases} \lambda - 1 & \text{if} \quad m_j = 1 \\ 0 & \text{if} \quad m_j = 0 \end{cases}$$

8. Aggregate labor share ω (equal to the normalized wage rate in the economy):

$$\omega = 1 - \mu \frac{(\lambda - 1)}{\lambda}. \tag{A.6}$$

9. Stock market value of firms that are in state $m_i \in \{-1, 0, 1\}$, which are denoted by v_{m_i}:

$$\begin{aligned} \rho v_1 &= \max_{x_1} \left\{ \left(1 - \frac{1}{\lambda}\right) + x_1 \left[v_1 - v_1\right] + (x_{-1} + \delta) \left[v_0 - v_1\right] \right\} \\ \rho v_{-1} &= \max_{x_{-1}} \left\{ -\frac{x_{-1}^2}{2} + (x_{-1} + \delta) \left[v_0 - v_{-1}\right] \right\} \\ \rho v_0 &= \max_{x_0} \left\{ -\frac{x_0^2}{2} + x_0 \left[v_1 - v_0\right] + x_0 \left[v_{-1} - v_0\right] \right\}. \end{aligned}$$

10. Optimal innovation decisions of leaders, neck-and-neck firms and followers:

$$\begin{aligned} x_1 &= 0 \\ x_0 &= v_1 - v_0 \\ x_{-1} &= v_0 - v_{-1}. \end{aligned} \tag{A.7}$$

11. The law of motion for μ:

$$\dot{\mu} = -\mu \left(x_{-1} + \delta\right) + (1 - \mu) 2 x_0. \tag{A.8}$$

B Proof of Proposition 1

In this environment firm values can be written as

$$\begin{aligned} r v_{-1} &= -\frac{x_{-1}^2}{2} + x_{-1} \left[v_0 - v_{-\bar{m}}\right] \\ r v_0 &= -\frac{x_0^2}{2} + x_0 \left[v_1 - v_0\right] + x_0 \left[v_{-1} - v_0\right] \\ r v_1 &= 2\Xi - \frac{x_1^2}{2} + x_1 \left[v_1 - v_1\right] + x_{-1} \left[v_0 - v_1\right] \end{aligned}$$

Here, $v_m \equiv V_m q^{-1}$. Note that, in this setting, the leader does not innovate, i.e., $x_1 = 0$. Now we show $x_0 > x_{-1} > 0$.

1. $v_1 > v_0$: Assume not such that $v_0 \geq= v_1$. Then $[v_1 - v_0] \leq 0$, and $x_0 = 0$. This implies

$v_0 = 0 \geq v_1$. But $v_0 = 0$ would mean $rv_1 = 2\Xi - x_{-1}v_1$ and thus $v_1 > 0$, a contradiction. Therefore $x_0 > 0$.

2. $v_0 > v_{-1}$: Assume not such that $v_{-1} \geq v_0$. Then $x_{-1} = 0$ implying that $v_{-1} = 0 \geq v_0$. This is possible only if $x_0 = 0$. But since $v_1 > v_0$ as shown above, $x_0 > 0$, a contradiction. Therefore $x_{-1} > 0$.

3. $[v_1 - v_0] > [v_0 - v_{-1}]$: Assume not such that $[v_0 - v_{-1}] \geq [v_1 - v_0]$. This means $v_0 < 0$ unless $x_0 = 0$. If $v_0 < 0$, it is a contradiction by step 2. If $x_0 = 0$ meaning that $v_0 = 0$ it is a contradiction by step 1. Therefore $[v_1 - v_0] > [v_0 - v_{-1}]$ and $x_0 > x_{-1} > x_1 = 0$.